Lateral Force Induced by Rectangular Surcharge Loads on a Cross-Anisotropic Backfill

Cheng-Der Wang

Abstract: This article presents approximate but analytical-based solutions for computing the lateral force (force per unit length) and centroid location induced by horizontal and vertical surcharge surface loads resting on a cross-anisotropic backfill. The surcharge loading types include: point load, finite line load, and uniform rectangular area load. The planes of cross-anisotropy are assumed to be parallel to the ground surface of the backfill. Although the presented solutions have never been proposed in existing literature, they can be derived by integrating the lateral stress solutions recently addressed by the author. It is clear that the type and degree of geomaterial anisotropy, loading distances from the retaining wall, and loading types significantly influence the derived solutions. An example is given for practical applications to illustrate the type and degree of soil anisotropy, as well as the loading types on the lateral force and centroid location in the isotropic/cross-anisotropic backfills caused by the horizontal and vertical uniform rectangular area loads. The results show that both the lateral force and centroid location in a cross-anisotropic backfill are quite different from those in an isotropic one. The derived solutions can be added to other lateral pressures, such as earth or water pressure, which are necessary in the stability and structural analysis of a retaining wall. In addition, they can be utilized to simulate more realistic conditions than the surcharge strip loading in geotechnical engineering for the backfill geomaterials are cross-anisotropic.

DOI: 10.1061/(ASCE)1090-0241(2007)133:10(1259)

CE Database subject headings: Lateral stress; Lateral loads; Vertical loads; Horizontal loads; Anisotropy; Backfills.

Introduction

Retaining structures usually carry earth pressure, water pressure, and frequently, surcharge pressure on their backfills. Surcharge loads might be anything from truck wheels, railway tracks, highway pavements, or to the foundations of adjacent buildings. These loading types could be modeled by point loads, line loads, strip loads, or area loads. It was found from a series of experiments performed at the Iowa Engineering Experiment Station (Spangler 1936; Spangler 1938a; Spangler 1938b; Spangler and Mickle 1956; Spangler and Handy 1982), and using the theory of elasticity method (Boussinesq 1885; Mindlin 1936) that when surcharge loads on the backfill were close enough to the retaining wall, additional lateral stress was produced. Traditionally, backfill material has been assumed to be a homogeneous, linearly elastic, and isotropic continuum. Nevertheless, various investigators (i.e., Michell 1900; Barden 1963; Pickering 1970; Gerrard and Wardle 1973; Gibson 1974; Gazetas 1982; Wang 2003; Abelev and Lade 2004) recognize the fact that many natural soils are deposited by geological sedimentation over a long period of time; therefore, they are not generally isotropic materials, but rather anisotropic, since the properties in the horizontal and vertical planes are different. That is, better results can be obtained by considering the anisotropic deformability. In this work, lateral force and its centroid location induced by three surcharge loads on a cross-anisotropic backfill are addressed.

Recently, Wang (2005; 2007) presented approximate but analytical-based solutions for calculating the lateral stress (Wang 2005), lateral force, and centroid location (Wang 2007) due to horizontal and vertical surcharge surface strip loads acting on a cross-anisotropic backfill. These surcharge loading types included a horizontal/vertical point load, a horizontal/vertical infinite line load, a horizontal/vertical uniform strip load, a horizontal/vertical upward linear-varying strip load, a horizontal/vertical upward nonlinear-varying strip load, a horizontal/vertical downward linear-varying strip load, and a horizontal/vertical downward nonlinear-varying strip load. However, in the case of a backfill resulting from an arbitrarily shaped load, it is well known that the area could be divided into several regularly shaped subareas, such as rectangles or triangles. Therefore, the strip loading solution for the induced lateral stress, lateral force, and centroid location might not be suitable. For this reason, Wang (unpublished material, 2006) gave the lateral stress solution for a cross-anisotropic backfill subjected to the aforementioned loadings, but distributed over a rectangular region. A part of Wang’s solutions (unpublished material, 2006), lists loading Case A: A horizontal/vertical point load (P/Q, force); Case B: A horizontal/vertical infinite line load (P'/Q', force per unit length); Case C: A horizontal/vertical uniform rectangular area load (P'/Q', force per unit area) with respect to the lateral stress solution is provided in Appendix I. Although the presentation of the lateral stress solution is clear and concise, they cannot easily applied to calculate the lateral force (force per unit length), since the computation of an irregular lateral stress area behind a retaining wall is not as simple as esti-
mating an earth or water pressure area. Hence, Jarquio (1981) derived the exact solution for determining the lateral force and centroid location (measuring from the wall top) for unyielding walls owing to a vertical uniform strip load exerting on an isotropic backfill. Then, Steenfelt and Hansen (1983) suggested that the elastic solution based on Boussinesq’s half-space (1885) was only reasonable for unyielding structures (Georgiadis and Anagnostopoulos 1998). Lately, Yildiz (2003) investigated the lateral pressures on unyielding walls due to surface strip loads by considering the nonlinear stress-strain behavior of the soil using the commercial finite element code, PLAXIS. Data obtained from the finite element analyses were used to train neural networks in order to acquire a solution to assess the lateral force and its point of application. To the best of the author’s knowledge, no analytical solutions have been proposed for the lateral force and centroid location generated by loading Cases A–C for a homogeneous, linearly elastic, and cross-anisotropic backfill. In deriving the presented solutions, the retaining wall is taken to be vertical with the horizontal backfill, meaning that the planes of cross-anisotropy are parallel to the horizontal ground surface. Moreover, two simplifying assumptions are made in this work: (1) the wall does not move; and (2) the wall is perfectly smooth (there is no shear stress between the wall and the soil). These two assumptions would imply that the induced lateral stress on a retaining wall is the same as the horizontal stress in an elastic half-space induced by two loads of equal magnitude acting on the surface (Fang 1991). However, in a real situation, a large stress concentration might be developed around the lower corner of the retaining wall in contact with the backfill. In addition, the theory of elasticity utilized in this study does not consider the strength and the variation of the stiffness of the soil with different stress states. Also, the assumption of a perfectly smooth wall is restrictive, and would be limited to the applicability of the elasticity method in practical applications. Nevertheless, as mentioned above, a series of experiments conducted at the Iowa Engineering Experiment Station (Spangler 1936; Spangler 1938a; Spangler 1938b; Spangler and Mickle 1956; Spangler and Handy 1982) and by Terzaghi (1954), confirmed the fact that doubling the horizontal stress in an elastic half-space could provide a good approximation to measured values of earth pressures on retaining walls (Fang 1991).

Another interesting and alternative approach by Constantinou and Gazetas (1986) by using a systematic relaxation-of-constraints technique of Hetenyi (1960, 1970), a plane-strain solution was derived for stresses in an elastic orthotropic quarter plane loaded by a vertical line load located at a distance from the edge. Further, Constantinou and Greenleaf (1987) yielded the solutions of stresses in the same plane due to a horizontal line load at a distance from the edge, a vertical uniform strip load at the edge, and at a distance from the edge. Their solutions were also specialized to the case of cross-anisotropic material with both real and complex-valued roots of the characteristic relation.

In this article, integrating the lateral stress of loading Cases A–C (Appendix I) with respect to z direction could obtain the closed-form solutions for the lateral force and centroid location induced by the horizontal and vertical surcharge loads (point load, finite line load, and uniform rectangular area load) on a cross-anisotropic backfill. The proposed approximate but analytical-based solutions could not only complete the full analysis of a retaining wall structure with a cross-anisotropic backfill subjected to a horizontal/vertical uniform rectangular load, but could also be extended to calculate the lateral force and centroid location due to an arbitrarily shaped loaded area by superposition. These solutions indicate that the type and degree of geomaterial anisotropy, loading distances from the retaining walls, dimensions of the loading area, and horizontal/vertical loading types deeply impact the induced lateral force and centroid location. An example is given for practical applications to illustrate the type and degree of soil anisotropy, and loading types on the lateral force and centroid location in the isotropic/cross-anisotropic backfills caused by a horizontal/vertical uniform rectangular area load.

**Lateral Stress Induced by a Horizontal/Vertical Point Load, Finite Line Load, and Uniform Rectangular Area Load**

Point load solutions in exact closed-form have always played an important role in applied mechanics. For the displacements and stresses in cross-anisotropic media subjected to a point load, analytical solutions have been presented by numerous investigators (i.e., as shown in the classical textbook of Lekhnitskii (1963); Liao and Wang 1998; Wang and Liao 1999). In this article, the solutions for lateral stress due to a horizontal/vertical surcharge point load, finite line load, and uniform rectangular area load on a cross-anisotropic backfill were derived by Wang (unpublished material, 2006) by integrating the point load solution of Wang and Liao (1999). A detailed deriving approach for solving the lateral stress, \( \sigma^x \), subjected to a horizontal surface point load \( P \), and a vertical one, \( Q \) (loading Case A), located at a horizontal distance in the \( x \)-axis from the retaining wall, \( a \), and at a horizontal distance in the \( y \)-axis from the retaining wall, \( c \) (\( x=a, y=c, z=0 \)), with height \( H \), as depicted in Fig. 1(a), can be referred to Wang (unpublished material, 2006). In addition, integrating \( \sigma^x \) for a smooth, rigid retaining wall could produce the approximate but analytical-based solutions for lateral stress, \( \sigma^x \), and \( \sigma^y \), subjected to loading Cases B and C, respectively. A summary of the lateral stress solutions, including \( \sigma^x \), \( \sigma^y \), and \( \sigma^z \), is given in Appendix I.

In Appendix I, \( P_{x1}, P_{y1} \) [Eqs. (1)–(4)], \( d_{x1}, d_{y1} \) [Eqs. (5)–(8)], and \( \varepsilon_{x1}, \varepsilon_{y1} \) [Eqs. (9)–(12)] \( i=1, 2, \text{ and } 3 \) are defined as the stress elementary functions. Hence, the lateral force and centroid location solutions could be yielded by integrating the stress elementary functions. The deriving procedures are as follows.

**Lateral Force Induced by a Horizontal/Vertical Point Load, Finite Line Load, and Uniform Rectangular Area Load**

Retaining walls support backfill earth pressure, water pressure, and often the surcharge pressure in the field of geotechnical engineering. Practically, it is easy to compute the resultant force and location of the resultant force induced by the earth and water pressures. Nevertheless, from a previous study on lateral stress (Wang, unpublished material, 2006), it seems to be difficult to calculate an irregular lateral stress area as lateral stress solutions are influenced by several factors, such as the type and degree of geomaterial anisotropy, loading distances from the retaining walls, dimensions of the loading area, and different horizontal/vertical loading types. Hence, analytical solutions for lateral force and its centroid location generated by the presented loading Cases A–C are necessary.

In this work, lateral force and centroid location could be directly obtained by integrating the stress elementary functions of
each lateral stress solution (Appendix I). For example, the complete solution of lateral force (force per unit length), \( P_{h} \), induced by a horizontal/vertical point load, as shown in Fig. 1(a), could be derived by integrating the stress elementary functions, \( p_{s11}^{h}P_{sh} \) \((i=1, 2, \text{ and } 3)\) [Eqs. (1)–(4) in Appendix I], in the \( z \) direction between limits 0 and \( H \) (height of the retaining wall). That is, the expression of the lateral force, \( P_{h} \), is the same as that of \( \sigma_{h}^{x} \), except that \( p_{s11}^{h}P_{sh} \) \((i=1, 2, \text{ and } 3)\) should be replaced by force integral functions, \( p_{s11}^{h}P_{sh} \) \((i=1, 2, \text{ and } 3)\), respectively. The lateral force, \( P_{h} \), and related force integral functions, \( p_{s11}^{h}P_{sh} \) \((i=1, 2, \text{ and } 3)\) for loading Case A are presented in Appendix II. Similarly, solutions for the lateral force, \( P_{h}^{l} \) [finite line load, Fig. 1(b)] and \( P_{h}^{u} \) [uniform rectangular area load, Fig. 1(c)], and their related force integral functions for loading Cases B and C could be obtained. They are also presented in Appendix II. However, observing Appendix II, the force integral functions, \( e_{11} \) [Eq. (21)], \( e_{22} \) [Eq. (22)], \( e_{31} \) [Eq. (23)], and \( e_{45} \) [Eq. (24)] \((i=1, 2, \text{ and } 3)\) for loading Case C are all functions of \( k_{1}=a_{u}H, k_{2}=l/uH, k_{3}=c/uH, \) and \( k_{4}=w/uH \) \((i=1, 2, \text{ and } 3)\). Thus, if the five elastic engineering constants \( E, E', v, v', \) and \( G' \) are given, then the characteristic root \( u_{i} \) \((i=1, 2, \text{ and } 3)\) can be calculated using the characteristic equation listed in Appendix I. Figs. 2–5 show the calculation charts of \( e_{11} \) [Eq. (21)], \( e_{22} \) [Eq. (22)], \( e_{31} \) [Eq. (23)], and \( e_{45} \) [Eq. (24)] for computing the lateral force induced by loading Case C, respectively. In these figures, \( k_{1} \) and \( k_{3} \) are equal to 0, that means the horizontal/vertical uniform rectangular area load is acting nearby the retaining structures \((a=c=0)\). Consequently, for the rest of variable nondimensional factors, \( k_{2} \) and \( k_{4} \), in which \( k_{2} \) is in the range 0.01–10, and \( k_{4} \) is in the range 0.4–6. These four calculation charts could be utilized to estimate the induced lateral force by a horizontal/vertical uniform rectangular load in a conservative manner when computers or calculators are unavailable, but they are only suitable for the root type of the characteristic equation (Appendix I) belonging to Case 1 \((i.e., \text{where } t_{1} \text{ and } u_{1} \text{ are two real distinct roots})\). In other words, the proposed figures \((\text{Figs. 2–5})\) cannot be applied to calculate the lateral force for loading Case C when the root type belongs to Case 2 and Case 3 (Appendix I).

**Centroid Location Induced by a Horizontal/Vertical Point Load, Finite Line Load, and Uniform Rectangular Area Load**

Since the location of resultant lateral force is significant for the stability and structural analysis of a retaining wall, the centroid location solution for loading Cases A–C are derived and presented in Appendix III. For instance, the centroid location \( \bar{z} \) measuring from the top of a wall, induced by a horizontal point load (namely, the vertical-direction point force \( Q=0 \)) could be acquired by multiplying \( z \) with \( \sigma_{h}^{x} \) (lateral stress resulting from a horizontal point load, as shown in Appendix I), and integrating with the limits of \( z \) from 0 to \( H \), then dividing by the yielded lateral force, \( P_{h} \) \((\text{when } Q=0, \text{as expressed in Appendix II})\). The same process holds for the other centroid location, \( \bar{z}^{l} \), when the horizontal-direction point force \( P=0 \). Repeatedly, in accordance with Appendix I, the stress elementary functions, \( p_{s11}^{h}P_{sh} \) \((i=1, 2, \text{ and } 3)\) [Eqs. (1)–(4)] in \( \sigma_{h}^{x} \) are integrated and limits are evaluated. They are then defined as centroid integral functions, \( p_{s11}^{h}P_{sh} \) \((i=1, 2, \text{ and } 3)\). Hence, only centroid location solutions and related centroid integral functions for loading Cases A–C are presented in Appendix III. According to Appendix III, another set of calculation charts could be drawn from the centroid integral functions, \( e_{11} \) [Eq. (33)], \( e_{22} \) [Eq. (34)], \( e_{31} \) [Eq. (35)], and \( e_{45} \) [Eq. (36)] \((i=1, 2, \text{ and } 3)\) for computing the centroid location induced by loading Case C. They are plotted, respectively, in Figs. 6–9.

Fig. 10 demonstrates a flow chart that illustrates the use of the eight calculation charts \( e_{11} \) (Fig. 2), \( e_{22} \) (Fig. 3), \( e_{31} \) (Fig. 4), \( e_{45} \) (Fig. 5), and \( e_{11} \) (Fig. 6), \( e_{22} \) (Fig. 7), \( e_{31} \) (Fig. 8), and \( e_{45} \) (Fig.
for computing the lateral force and centroid location induced by the horizontal and vertical uniform surcharge rectangular area loads (loading Case C). Although these charts are given for five elastic engineering constants of a cross-anisotropic backfill belonging to Case 1, they can be an alternative tool for supplying results with reasonable accuracy.

The use of the presented formulae could save users time in calculating the lateral force and centroid location owing to sur-

charge surface loads involving a horizontal/vertical point load, finite line load, and uniform rectangular load, resting on a cross-
anisotropic backfill. Moreover, the derived solutions are identical to Wang’s horizontal and vertical strip loading solutions (Wang 2007) for a cross-anisotropic backfill in the case of $c=0$, and $w=\infty$. Additionally, they are in very good agreement with the isotropic solutions of Jarquio (1981) by using a limiting approach for the vertical uniform strip loading case ($c=0$ and $w=\infty$).

Fig. 2. Calculation charts of $\varepsilon_{1i}$ ($i=1, 2, 3$) from Eq. (21) for computing the lateral force induced by loading Case C

Fig. 3. Calculation charts of $\varepsilon_{2i}$ ($i=1, 2, 3$) from Eq. (22) for computing the lateral force induced by loading Case C

Fig. 4. Calculation charts of $\varepsilon_{3i}$ ($i=1, 2, 3$) from Eq. (23) for computing the lateral force induced by loading Case C

Fig. 5. Calculation charts of $\varepsilon_{4i}$ ($i=1, 2, 3$) from Eq. (24) for computing the lateral force induced by loading Case C
Practical Applications

An example for computing the lateral force and centroid location induced by a horizontal/vertical uniform rectangular load is illustrated in this section. The plane of a horizontal/vertical loaded area EFGH with a uniform intensity \( P_s/Q_s \) acting on the surface of an isotropic and cross-anisotropic backfill is shown in Fig. 11. Seven types of isotropic (Soil 1) and cross-anisotropic soils (Soils 2–7) are considered as the constituted backfill geomaterials. The influence of the degree of soil anisotropy, specified by the ratios \( E/E', v/v' \), and \( G/G' \) on the lateral force and centroid location is examined. Table 1 lists their elastic properties and root type. The selected domains of anisotropic variation basically follow the suggestion of Gazetas (1982) with \( E/E' \) ranging from 0.6 to 4 for clays, and as low as 0.2 for sands. Hence, they are hypothetically assumed that \( v/v' \) varying between 0.75–1.5, and \( E/E' \) and \( G/G' \) ranging between 0.15–1.5. The values of \( E \) and \( v \) adopted in Table 1 are 50 MPa and 0.3, respectively.

Based on Appendixes II and III, a Mathematica program was written to compute the lateral force and centroid location due to the proposed loading cases. However, in this section, a loaded...
area as depicted in Fig. 11 is illustrated as a practical example. In Fig. 11, \( a \) = horizontal and vertical uniform rectangular loads applied at a horizontal distance in the \( x \)-axis from the retaining wall, and \( l \) = length of the rectangular load. The normalized lateral force, \( P_h^{\text{a}}/P^a \) and \( P_v^{\text{a}}/Q^a \), and centroid location measuring from the top of the retaining wall, \( z_h^a \) and \( z_v^a \), for Soils 1–7 are calculated and plotted, respectively, in Figs. 12–15.

Fig. 12 presents the normalized lateral force \( P_h^{\text{a}}/P^a \) versus \( a/l \) (from 0 to 5) for Soils 1, 2, and 3 [Fig. 12(a)], Soils 1, 4, and 5 [Fig. 12(b)], and Soils 1, 6, and 7 [Fig. 12(c)] resulting from a horizontal uniform rectangular loaded area of Fig. 11. The variations of \( P_h^{\text{a}}/P^a \) for Soils 1–7 are within the 0–0.16 range, and except for Soils 2 and 6, they are slightly influenced by the type and degree of geomaterial anisotropy.

Fig. 13 depicts the effect of \( a/l \) generated by a vertical uniform rectangular load (Fig. 11) on the normalized lateral force \( P_v^{\text{a}}/Q^a \), especially for Soils 2 [Fig. 13(a)] and 6 [Fig. 13(c)], is markedly affected by the type and degree of backfill anisotropy. Fig. 14 exhibits the centroid location \( z_h^a \) (measuring from the wall top), for Soils 1–7 caused by a horizontal uniform rectangular loaded area of Fig. 11. The variations of \( z_h^a \) for Soils 1–7 are within −0.5–2.5. Again, they are slightly impacted by the type and degree of material anisotropy except for Soils 2 and 6.

Fig. 11. The plane of a horizontal/vertical uniform surcharge loaded area EFGH acting on the surface of the isotropic/cross-anisotropic backfills.

Conclusions

The closed-form solutions derived by Wang (unpublished material, 2006) for the lateral stress induced by a horizontal/vertical

Table 1. Elastic Properties and Root Type for the Isotropic/Cross-Anisotropic Soils

<table>
<thead>
<tr>
<th>Soil type</th>
<th>( E/E' )</th>
<th>( v/v' )</th>
<th>( G/G' )</th>
<th>Root type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Isotropy</td>
<td>1</td>
<td>1</td>
<td>Equal</td>
<td></td>
</tr>
<tr>
<td>2. Cross-anisotropy</td>
<td>0.15</td>
<td>1</td>
<td>1</td>
<td>Distinct</td>
</tr>
<tr>
<td>3. Cross-anisotropy</td>
<td>1.5</td>
<td>1</td>
<td>1</td>
<td>Complex</td>
</tr>
<tr>
<td>4. Cross-anisotropy</td>
<td>1</td>
<td>0.75</td>
<td>1</td>
<td>Complex</td>
</tr>
<tr>
<td>5. Cross-anisotropy</td>
<td>1</td>
<td>1.5</td>
<td>1</td>
<td>Distinct</td>
</tr>
<tr>
<td>6. Cross-anisotropy</td>
<td>1</td>
<td>1</td>
<td>0.15</td>
<td>Complex</td>
</tr>
<tr>
<td>7. Cross-anisotropy</td>
<td>1</td>
<td>1</td>
<td>1.5</td>
<td>Distinct</td>
</tr>
</tbody>
</table>

Note: \( E=50 \text{ MPa} \) and \( v=0.3 \) are adopted.
point load, finite line load, and uniform rectangular area load, exerting on a cross-anisotropic backfill can be expressed in terms of stress elementary functions. Integration of the stress elementary functions, solutions of lateral force and centroid location’s integral functions can be yielded. These integral functions can be applied to construct the calculation charts for computing the lateral force and centroid location resulting from the horizontal/vertical uniform rectangular load when computers or calculators are unavailable. Nevertheless, these charts (Figs. 2–9) are only suitable for the root type of the characteristic equation belonging to Case 1 (i.e., where \( u_1 \) and \( u_2 \) are two real distinct roots).

Based on the results of a parametric study by a practical example, it is found that the lateral force and centroid location are both intensely affected by the type and degree of soil anisotropy (Soils 1–7), as well as different loading types (horizontal and vertical). The calculation of the induced lateral force and centroid location by a horizontal/vertical point load, finite line load, and uniform rectangular load in an isotropic/cross-anisotropic backfill is fast and correct, since the presentation of the derived solutions is clear and concise. These approximate but analytical-based solutions can be utilized to simulate more realistic conditions than the surcharge strip loading in geotechnical engineering for the backfill geomaterials are cross-anisotropic. Additionally, the derived solutions can be added to other lateral pressures, such as

---

Fig. 12. Effect of \( a/l \) on lateral force caused by a horizontal uniform rectangular loaded area of Fig. 11 for: (a) soils 1, 2, and 3; (b) soils 1, 4, and 5; and (c) soils 1, 6, and 7
earth or water pressure, necessary for the stability and structural analysis of a retaining wall. The proposed formulae are identical with Wang’s horizontal and vertical uniform strip loading solutions (Wang 2007) for a cross-anisotropic backfill when $c=0$, and $w=\infty$. However, by limiting a procedure, they are also in good agreement with Jarquio’s vertical uniform strip loading solutions (Jarquio 1981) for an isotropic backfill. Moreover, these solutions can be extended to compute the lateral force and centroid location subjected to a horizontal/vertical uniform arbitrarily shaped area load by superposition. Regarding the solutions of the lateral force and centroid location resulting from a horizontal/vertical linear-varying and nonlinear-varying rectangular area loads can also be explored. With these solutions, the lateral force and centroid location owing to any conceivable irregular area and distributed load can be fully analyzed. The results of these investigations will be addressed in forthcoming works.

**Acknowledgments**

The writer would like to thank the valuable comments of Editor and reviewers, and special thanks to the loving supports of his family during this work.
Fig. 14. Effect of $a/l$ on centroid location caused by a horizontal uniform rectangular loaded area of Fig. 11 for: (a) soils 1, 2, and 3; (b) soils 1, 4, and 5; and (c) soils 1, 6, and 7
Fig. 15. Effect of \( \ell / l \) on centroid location caused by a vertical uniform rectangular loaded area of Fig. 11 for: (a) soils 1, 2, and 3; (b) soils 1, 4, and 5; and (c) soils 1, 6, and 7
## Appendix I. Solutions of Lateral Stress due to Loading Cases A–C

<table>
<thead>
<tr>
<th>Type of loading</th>
<th>Lateral stress solutions</th>
<th>Stress elementary functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>( \sigma_{pi}^a = \frac{p}{2\pi} \left[ k(m_2u_1-m_1u_2) \left( A_{44}(u_1^2+p_{s31}-u_2^2+p_{s32}) \right) - 2A_{66} \left( \frac{u_1+m_1+u_2}{m_2+u_2} \right) \right] )</td>
<td>Eqs. (1)–(4)</td>
</tr>
<tr>
<td>Case B</td>
<td>( \sigma_{pi}^b = \frac{p'<em>{iy}}{2\pi} \left[ k(m_2u_1-m_1u_2) \left( A</em>{44}(u_1^2+d_{s1i}-u_2^2+d_{s2i}) \right) - 2A_{66} \left( \frac{u_1+m_1+u_2}{m_2+u_2} \right) \right] )</td>
<td>Eqs. (5)–(8)</td>
</tr>
<tr>
<td>Case C</td>
<td>( \sigma_{pi}^c = \frac{p'<em>{iz}}{2\pi} \left[ k(m_2u_1-m_1u_2) \left( A</em>{44}(u_1^2+e_{s1i}-u_2^2+e_{s2i}) \right) - 2A_{66} \left( \frac{u_1+m_1+u_2}{m_2+u_2} \right) \right] )</td>
<td>Eqs. (9)–(12)</td>
</tr>
</tbody>
</table>

where

\[
p_{sli} = \frac{x+a}{R_i^3} \quad (1)
\]

\[
p_{s2i} = \frac{x+a}{R_i^3} - \frac{3(x+a)}{R_i(R_i+z_i)} - \frac{R_i}{R_i^3(R_i+z_i)^3} \quad (2)
\]

\[
p_{s3i} = \frac{z_i}{R_i^3} \quad (3)
\]

\[
p_{s4i} = \frac{1}{R_i(R_i+z_i)} - \frac{(x+a)^3(2R_i+z_i)}{R_i^3(R_i+z_i)^2} \quad (4)
\]

\[
d_{s1i} = \frac{c}{[(x+a)^2+z_i]^2} - \frac{c+l}{\sqrt{(x+a)^2+z_i}^2} = \frac{c+l}{\sqrt{(x+a)^2+(c+l)^2+z_i}^2} \quad (5)
\]

\[
d_{s2i} = -(x+a) \left\{ \frac{c}{[(x+a)^2+z_i]^2} - \frac{c+l}{\sqrt{(x+a)^2+z_i}^2} \right\} \quad (6)
\]

\[
d_{s3i} = -\frac{z_i}{[(x+a)^2+z_i]^2} - \frac{c+l}{\sqrt{(x+a)^2+(c+l)^2+z_i}^2} \quad (7)
\]

\[
d_{s4i} = \frac{c}{(x+a)^2+z_i^2} - \frac{c+l}{(x+a)^2+(c+l)^2+z_i^2} - \frac{z_i}{[(x+a)^2+z_i]^2} - \frac{c+l}{\sqrt{(x+a)^2+(c+l)^2+z_i}^2} \quad (8)
\]

\[
e_{s1i} = \ln \left( \frac{\sqrt{n_i}+n_{i1}+n_{i2}}{n_i^2+n_{i1}+1+n_{i2}} \right) - \ln \left( \frac{\sqrt{n_i}+n_{i1}+n_{i2}}{n_i^2+n_{i1}+1+n_{i2}} \right) \quad (9)
\]

\[
JOURNAL OF GEOTECHNICAL AND GEOENVIRONMENTAL ENGINEERING © ASCE / OCTOBER 2007 / 1269
\]
where

\[ R_i = \sqrt{(x+a)^2 + (y+c)^2 + z_i^2}, \]

\[ n_i = a / z_i, n_{2i} = b / z_i, n_{3i} = c / z_i, n_{4i} = w / z_i, z_i = u_i (i = 1, 2, \text{and } 3). \]

• The definition of \( a, b, c, w, P / Q \) (force), \( P / Q' \) (force per unit length), and \( P / Q'' \) (force per unit area) can be referred to Fig. 1 and the Notation section.

* \( x, y \) denote the desired horizontal positions; and \( z \) =vertical distance from point to load in a Cartesian coordinate system.

\[ A_{ij} (i,j=1–6) \]

denote the elastic moduli or elasticity constants of the backfill, in which \( A_{11} = E_1 (1 - \nu_1^2) (1 + \nu) / (1 - \nu^2) \) represents the Young’s modulus in the horizontal direction, and \( A_{33} = E_3 (1 - \nu_3^2) (1 + \nu) / (1 - \nu^2) \) denotes the shear modulus in the vertical plane. \( \nu \) represents the Poisson’s ratio for the effect of vertical stress on horizontal elastic strain, \( \nu \) represents the Poisson’s ratio for the effect of vertical stress on horizontal strain, \( G \) represents the shear modulus in the vertical plane.

\[ u_3 = \sqrt{A_{66} / A_{44}}, \]

\( u_1 \) and \( u_2 \) denote the roots of the characteristic equation: \( x^2 - u x^2 + t = 0, \)

in which \( t = A_{11} A_{33} - A_{13} A_{31} (A_{12} + A_{44}) / A_{33} A_{44}, \)

\( A_{33} = A_{11} / A_{33}, \)

and it can be categorized into three cases: (1) Case 1. \( u_{1,2} = \pm \sqrt{1 / 2 \left( s \pm \sqrt{s^2 - 4 t} \right)} \) has two real distinct roots when \( s^2 - 4 t > 0, \)

(2) Case 2. \( u_{1,2} = \pm \sqrt{s \pm \sqrt{s^2 - 4 t}} \) has two complex conjugate roots (where \( s \neq 0 \) when \( s^2 - 4 t < 0, \)

\( k = (A_{13} + A_{44}) / A_{33} A_{44} (u_1^2 - u_2^2), m_i = (A_{13} + A_{44}) u_i / A_{33} (u_i^2 - A_{44}) = (A_{11} - A_{44} u_i^2) / (A_{13} + A_{44}) u_i \) (i = 1, 2, \( \text{and } 3). \)

### Appendix II. Solutions of Lateral Force due to Loading Cases A–C

<table>
<thead>
<tr>
<th>Type of loading</th>
<th>Lateral force solutions</th>
<th>Force integral functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>[ p_h^* = 2 \pi \left( k_m (u_1 - u_2) / m_1 m_2 (u_1 - u_2) \right) A_{44} (u_1^2 - u_2^2) / A_{44} (u_1^2 - u_2^2) + 2 A_{66} / m_1 + u_1 ] + [ 2 \pi Q / u_1 - u_2 A_{44} (u_1^2 - u_2^2) / A_{44} (u_1^2 - u_2^2) + 2 A_{66} / m_1 + u_1 ]</td>
<td>Eqs. (13)–(16)</td>
</tr>
</tbody>
</table>

Case B

\[ p_h^* = 2 \pi \left( k_m (u_1 - u_2) / m_1 m_2 (u_1 - u_2) \right) A_{44} (u_1^2 - u_2^2) / A_{44} (u_1^2 - u_2^2) + 2 A_{66} / m_1 + u_1 \] + \[ 2 \pi Q / u_1 - u_2 A_{44} (u_1^2 - u_2^2) / A_{44} (u_1^2 - u_2^2) + 2 A_{66} / m_1 + u_1 \] | Eqs. (17)–(20) |

Case C

\[ p_h^* = 2 \pi \left( k_m (u_1 - u_2) / m_1 m_2 (u_1 - u_2) \right) A_{44} (u_1^2 - u_2^2) / A_{44} (u_1^2 - u_2^2) + 2 A_{66} / m_1 + u_1 \] + \[ 2 \pi Q / u_1 - u_2 A_{44} (u_1^2 - u_2^2) / A_{44} (u_1^2 - u_2^2) + 2 A_{66} / m_1 + u_1 \] | Eq. (21) for Fig. 2 | Eq. (22) for Fig. 3 | Eq. (23) for Fig. 4 | Eq. (24) for Fig. 5 |

where

\[ p_{11} = \frac{t}{(s + a)^2 + (y + c)^2 + (x + a)^2 + (y + c)^2 + (u_i H)^2} \]
\[ \begin{align*}
 p_{t_1i} &= -(x + a)\frac{[(x + a)^2 - 3(y + c)^2]}{[(x + a)^2 + (y + c)^2]}[uH - \sqrt{(x + a)^2 + (y + c)^2 + (uH)^2}] - \frac{(y + c)^2}{[(x + a)^2 + (y + c)^2]^2[(x + a)^2 + (y + c)^2 + (uH)^2]} \\
 p_{t_2i} &= \frac{1}{u_i} \left[ \frac{1}{\sqrt{(x + a)^2 + (y + c)^2}} - \frac{1}{\sqrt{(x + a)^2 + (y + c)^2} + (uH)^2} \right] \\
 p_{t_3i} &= \frac{1}{u_i(\sqrt{(x + a)^2 + (y + c)^2})^2} \left\{ - \frac{u_iH(\sqrt{(x + a)^2 - (y + c)^2})}{(x + a)^2 + (y + c)^2} + \frac{\sqrt{(x + a)^2 + (y + c)^2} + (uH)^2}{u_iH(\sqrt{(x + a)^2 + (y + c)^2})} \right\} + \frac{\sqrt{(x + a)^2 + (y + c)^2} + (uH)^2}{u_iH(\sqrt{(x + a)^2 + (y + c)^2})} \\
 d_{t_1i} &= -\frac{1}{u_i} \left[ \tan^{-1} \frac{cuH}{(x + a)(\sqrt{(x + a)^2 + c^2} + (uH)^2)} - \tan^{-1} \frac{(c + w)uH}{(x + a)(\sqrt{(x + a)^2 + (c + w)^2} + (uH)^2)} \right] \\
 d_{t_2i} &= -(x + a)H \left[ \frac{c}{(x + a)^2 + c^2}(\sqrt{(x + a)^2 + c^2} + (uH)^2) - \frac{c + w}{(x + a)(\sqrt{(x + a)^2 + (c + w)^2} + (uH)^2)} \right] \\
 d_{t_3i} &= \frac{1}{u_i} \ln \left( \frac{c + \sqrt{(x + a)^2 + c^2} + (uH)^2}{c + \sqrt{(x + a)^2 + c^2}} \right) - \ln \left( \frac{c + w + \sqrt{(x + a)^2 + (c + w)^2} + (uH)^2}{c + w + \sqrt{(x + a)^2 + (c + w)^2}} \right) \\
 e_{t_1i} &= k_{t_1}(T_{t_1} - T_{t_2}) - (k_{t_1} + k_{t_3})(T_{t_3} - T_{t_4}) - L_{t_1} - k_{t_2}(L_{t_2} - L_{t_3}) + (k_{t_3} + k_{t_4})(L_{t_1} - L_{t_0}) \\
 e_{t_2i} &= -\frac{k_{t_1}}{2} \left\{ \frac{\sqrt{k_{t_1}^2 + k_{t_2}^2} + 1}{k_{t_1}^2 + k_{t_2}^2} - \frac{\sqrt{(k_{t_1} + k_{t_2})^2 + k_{t_3}^2} + 1}{(k_{t_1} + k_{t_2})^2 + k_{t_3}^2} \right\} + \frac{k_{t_3}^2 + (k_{t_3} + k_{t_4})^2 + 1}{k_{t_3}^2 + (k_{t_3} + k_{t_4})^2} - \frac{(k_{t_1} + k_{t_2})^2 + (k_{t_3} + k_{t_4})^2 + 1}{(k_{t_1} + k_{t_2})^2 + (k_{t_3} + k_{t_4})^2} \\
 e_{t_3i} &= T_{t_1} - T_{t_2} - T_{t_3} - T_{t_4} - k_{t_1}(L_{t_1} - L_{t_0}) + (k_{t_1} + k_{t_2})(L_{t_1} - L_{t_0}) - k_{t_3}(L_{t_1} - L_{t_2}) + (k_{t_3} + k_{t_4})(L_{t_3} - L_{t_4}) \\
 e_{t_4i} &= -(T_{t_1} - T_{t_2} - T_{t_3} + T_{t_4} + T_{t_1} - T_{t_4}) + T_{t_1}(L_{t_1} - L_{t_0}) - k_{t_1}(L_{t_1} - L_{t_0}) + (k_{t_1} + k_{t_2})(L_{t_2} - L_{t_1}) (i = 1, 2, 3) \text{ } H \text{ denotes the height of the retaining wall; } T_{t_1} - T_{t_2} \text{ and } L_{t_1} - L_{t_4} \text{ (i = 1, 2, and 3) are expressed as} \\
 T_{t_1i} &= \tan^{-1} \frac{k_{t_1}}{k_{t_1}^2 + k_{t_2}^2 + 1} \\
 T_{t_2i} &= \tan^{-1} \frac{k_{t_3} + k_{t_4}}{k_{t_1}^2 + (k_{t_3} + k_{t_4})^2 + 1} \\
 T_{t_3i} &= \tan^{-1} \frac{k_{t_3}}{(k_{t_1} + k_{t_2})^2 + k_{t_3}^2 + 1} \\
 \right. \end{align*} \]
\[ T_{4i} = \tan^{-1} \frac{k_{3i} + k_{4i}}{(k_{3i} + k_{4i}) \sqrt{(k_{1i} + k_{2i})^2 + (k_{3i} + k_{4i})^2 + 1}} \]

\[ T_{5i} = \tan^{-1} \frac{k_{1i}k_{3i}}{\sqrt{k_{1i}^2 + k_{3i}^2} + 1} \]

\[ T_{6i} = \tan^{-1} \frac{k_{3i}(k_{1i} + k_{2i})}{\sqrt{(k_{1i} + k_{2i})^2 + k_{3i}^2} + 1} \]

\[ T_{7i} = \tan^{-1} \frac{k_{1i}(k_{3i} + k_{4i})}{\sqrt{k_{1i}^2 + (k_{3i} + k_{4i})^2} + 1} \]

\[ T_{8i} = \tan^{-1} \frac{(k_{1i} + k_{2i})(k_{3i} + k_{4i})}{\sqrt{(k_{1i} + k_{2i})^2 + (k_{3i} + k_{4i})^2} + 1} \]

\[ T_{9i} = \tan^{-1} \frac{k_{3i} + k_{4i}}{k_{3i}} \]

\[ T_{10i} = \tan^{-1} \frac{k_{1i} + k_{2i}}{k_{3i}} \]

\[ T_{11i} = \tan^{-1} \frac{k_{1i}}{k_{3i} + k_{4i}} \]

\[ T_{12i} = \tan^{-1} \frac{k_{1i} + k_{2i}}{k_{3i} + k_{4i}} \]

\[ L_{1i} = \ln \left| \frac{\sqrt{(k_{1i} + k_{2i})^2 + k_{3i}^2} + 1 + k_{3i}}{\sqrt{k_{1i}^2 + k_{3i}^2} + 1 + k_{3i}} \right| \]

\[ L_{2j} = \ln \left| \frac{\sqrt{(k_{1i} + k_{2i})^2 + (k_{3i} + k_{4i})^2} + 1 + k_{3i} + k_{4i}}{\sqrt{k_{1i}^2 + (k_{3i} + k_{4i})^2} + 1 + k_{3i} + k_{4i}} \right| \]

\[ L_{3i} = \ln \left| \frac{\sqrt{(k_{1i} + k_{2i})^2 + k_{3i}^2} + 1}{\sqrt{k_{1i}^2 + k_{3i}^2} + 1} \right| \]

\[ L_{4i} = \ln \left| \frac{\sqrt{(k_{1i} + k_{2i})^2 + k_{3i}^2} + 1 + 1}{\sqrt{k_{1i}^2 + k_{3i}^2} + 1 + 1} \right| \]

\[ L_{5i} = \ln \left| \frac{\sqrt{(k_{1i} + k_{2i})^2} + (k_{3i} + k_{4i})^2}{\sqrt{k_{1i}^2} + (k_{3i} + k_{4i})^2} \right| \]

\[ L_{6i} = \ln \left| \frac{\sqrt{(k_{1i} + k_{2i})^2} + (k_{3i} + k_{4i})^2 + 1 + 1}{\sqrt{k_{1i}^2} + (k_{3i} + k_{4i})^2 + 1 + 1} \right| \]

\[ L_{7i} = \ln \left| \frac{\sqrt{k_{1i}^2 + k_{3i}^2} + 1 + k_{3i}}{\sqrt{k_{1i}^2 + k_{3i}^2} + k_{3i}} \right| \]

\[ L_{8i} = \ln \left| \frac{\sqrt{k_{1i}^2 + (k_{3i} + k_{4i})^2} + 1 + k_{3i} + k_{4i}}{\sqrt{k_{1i}^2 + (k_{3i} + k_{4i})^2} + k_{3i} + k_{4i}} \right| \]

\[ L_{9i} = \ln \left| \frac{\sqrt{(k_{1i} + k_{2i})^2} + k_{3i}^2 + 1 + k_{3i}}{\sqrt{(k_{1i} + k_{2i})^2} + k_{3i}^2 + k_{3i}} \right| \]

\[ L_{10i} = \ln \left| \frac{\sqrt{(k_{1i} + k_{2i})^2} + (k_{3i} + k_{4i})^2 + 1 + k_{3i} + k_{4i}}{\sqrt{(k_{1i} + k_{2i})^2} + (k_{3i} + k_{4i})^2 + k_{3i} + k_{4i}} \right| \]
\[ L_{11i} = \ln \left| \frac{k_{i1}^2 + k_{i2}^2 + 1 + k_{i1}}{k_{i1}^2 + k_{i3}^2 + k_{i1}} \right| \]
\[ L_{12i} = \ln \left| \frac{(k_{i1} + k_{i2})^2 + k_{i3}^2 + 1 + k_{i1} + k_{i2}}{(k_{i1} + k_{i2})^2 + k_{i3}^2 + k_{i1} + k_{i2}} \right| \]
\[ L_{13i} = \ln \left| \frac{k_{i1}^2 + (k_{i3} + k_{i2})^2 + 1 + k_{i1}}{k_{i1}^2 + (k_{i3} + k_{i2})^2 + k_{i1}} \right| \]
\[ L_{14i} = \ln \left| \frac{(k_{i1} + k_{i2})^2 + (k_{i3} + k_{i2})^2 + 1 + k_{i1} + k_{i2}}{(k_{i1} + k_{i2})^2 + (k_{i3} + k_{i2})^2 + k_{i1} + k_{i2}} \right| \]

### Appendix III. Solutions of Centroid Location due to Loading Cases A–C

<table>
<thead>
<tr>
<th>Type of loading</th>
<th>Centroid location solutions (measuring from the top of the retaining wall)</th>
<th>Centroid integral functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>[ z_{ih}^w = \frac{k(m_2u_1 - m_1u_2)}{m_1m_2(u_1 - u_2)} \left[ A_{44}(u_1^2 e_{11} - u_2^2 e_{12}) - 2A_{46} \left( \frac{u_1}{m_1 + u_1} * e_{21} - \frac{u_2}{m_2 + u_2} * e_{22} \right) \right] + 2u_1 e_{23} ]</td>
<td>Eqs. (25)–(28)</td>
</tr>
<tr>
<td></td>
<td>[ z_{ih}^p = \frac{k(m_2u_1 - m_1u_2)}{m_1m_2(u_1 - u_2)} \left[ A_{44}(u_1^2 d_{11} - u_2^2 d_{12}) - 2A_{46} \left( \frac{u_1}{m_1 + u_1} * d_{21} - \frac{u_2}{m_2 + u_2} * d_{22} \right) \right] + 2u_1 d_{23} ]</td>
<td>(when ( Q=0 ))</td>
</tr>
<tr>
<td></td>
<td>[ z_{ih}^q = \frac{k(m_2u_1 - m_1u_2)}{m_1m_2(u_1 - u_2)} \left[ A_{44}(u_1^2 e_{11} - u_2^2 e_{12}) - 2A_{46} \left( \frac{u_1}{m_1 + u_1} * e_{21} - \frac{u_2}{m_2 + u_2} * e_{22} \right) \right] + 2u_1 e_{23} ]</td>
<td>(when ( Q'=0 ))</td>
</tr>
<tr>
<td>Case B</td>
<td>[ z_{ih}^w = \frac{k(m_2u_1 - m_1u_2)}{m_1m_2(u_1 - u_2)} \left[ A_{44}(u_1^2 e_{11} - u_2^2 e_{12}) - 2A_{46} \left( \frac{u_1}{m_1 + u_1} * e_{21} - \frac{u_2}{m_2 + u_2} * e_{22} \right) \right] + 2u_1 e_{23} ]</td>
<td>Eqs. (29)–(32)</td>
</tr>
<tr>
<td></td>
<td>[ z_{ih}^p = \frac{k(m_2u_1 - m_1u_2)}{m_1m_2(u_1 - u_2)} \left[ A_{44}(u_1^2 d_{11} - u_2^2 d_{12}) - 2A_{46} \left( \frac{u_1}{m_1 + u_1} * d_{21} - \frac{u_2}{m_2 + u_2} * d_{22} \right) \right] + 2u_1 d_{23} ]</td>
<td>(when ( P=0 ))</td>
</tr>
<tr>
<td></td>
<td>[ z_{ih}^q = \frac{k(m_2u_1 - m_1u_2)}{m_1m_2(u_1 - u_2)} \left[ A_{44}(u_1^2 e_{11} - u_2^2 e_{12}) - 2A_{46} \left( \frac{u_1}{m_1 + u_1} * e_{21} - \frac{u_2}{m_2 + u_2} * e_{22} \right) \right] + 2u_1 e_{23} ]</td>
<td>(when ( P'=0 ))</td>
</tr>
<tr>
<td>Case C</td>
<td>[ z_{ih}^w = \frac{k(m_2u_1 - m_1u_2)}{m_1m_2(u_1 - u_2)} \left[ A_{44}(u_1^2 e_{11} - u_2^2 e_{12}) - 2A_{46} \left( \frac{u_1}{m_1 + u_1} * e_{21} - \frac{u_2}{m_2 + u_2} * e_{22} \right) \right] + 2u_1 e_{23} ]</td>
<td>Eq. (33) for Fig. 6</td>
</tr>
<tr>
<td></td>
<td>[ z_{ih}^p = \frac{k(m_2u_1 - m_1u_2)}{m_1m_2(u_1 - u_2)} \left[ A_{44}(u_1^2 e_{11} - u_2^2 e_{12}) - 2A_{46} \left( \frac{u_1}{m_1 + u_1} * e_{21} - \frac{u_2}{m_2 + u_2} * e_{22} \right) \right] + 2u_1 e_{23} ]</td>
<td>Eq. (34) for Fig. 7</td>
</tr>
<tr>
<td></td>
<td>[ z_{ih}^q = \frac{k(m_2u_1 - m_1u_2)}{m_1m_2(u_1 - u_2)} \left[ A_{44}(u_1^2 e_{11} - u_2^2 e_{12}) - 2A_{46} \left( \frac{u_1}{m_1 + u_1} * e_{21} - \frac{u_2}{m_2 + u_2} * e_{22} \right) \right] + 2u_1 e_{23} ]</td>
<td>Eq. (35) for Fig. 8</td>
</tr>
<tr>
<td></td>
<td>[ z_{ih}^r = \frac{k(m_2u_1 - m_1u_2)}{m_1m_2(u_1 - u_2)} \left[ A_{44}(u_1^2 e_{11} - u_2^2 e_{12}) - 2A_{46} \left( \frac{u_1}{m_1 + u_1} * e_{21} - \frac{u_2}{m_2 + u_2} * e_{22} \right) \right] + 2u_1 e_{23} ]</td>
<td>Eq. (36) for Fig. 9</td>
</tr>
</tbody>
</table>

where

\[ p_{31} = \frac{x + a}{u_1^2} \left( \frac{1}{\sqrt{(x + a)^2 + (y + c)^2}} - \frac{1}{\sqrt{(x + a)^2 + (y + c)^2 + (u_1H)^2}} \right) \]
\[
p_{zi} = \frac{(x + a)}{3u_i^2[(x + a)^2 + (y + c)^2]} \left\{ \frac{(x + a)^2}{(x + a)^2 + (y + c)^2} - \frac{2(u_i H)^2[(x + a)^2 - 3(y + c)^2]}{[(x + a)^2 + (y + c)^2]^2} + \frac{(x + a)^2 + (y + c)^2 + (u_i H)^2}{(x + a)^2 + (y + c)^2} \right\}
\]

\[
p_{zi} = -\frac{1}{u_i^2} \left\{ \frac{u_i H}{(x + a)^2 + (y + c)^2} \right\} \left\{ \ln \left[ \frac{u_i H + \sqrt{(x + a)^2 + (y + c)^2} + (u_i H)^2}{(x + a)^2 + (y + c)^2} \right] \right\}
\]

\[
d_{zi} = \frac{c(x + a)}{3a_i^2(\sqrt{(x + a)^2 + c^2})} \left\{ \frac{2(u_i H)^3}{(x + a)^2 + (y + c)^2 + c^3} - 1 + \left[ \frac{(x + a)^2 + c^2 - 2(x + a)^2 + c^2 + (u_i H)^2}{(x + a)^2 + c^3} \right] \right\}
\]

\[
d_{zi} = -\frac{1}{u_i^2} \left\{ \frac{u_i H}{\sqrt{(x + a)^2 + c^2} + (u_i H)^2} \right\} \left\{ \ln \left[ \frac{u_i H + \sqrt{(x + a)^2 + (c + w)^2 + (u_i H)^2}}{(x + a)^2 + (c + w)^2} \right] \right\}
\]

\[
e_{zi} = \frac{1}{2} \left\{ k_{3i} \left[ \sqrt{k_{1i}^2 + k_{2i}^2} - \sqrt{(k_{1i} + k_{3i} + k_{4i})^2 + k_{3i}^2 + 1} \right] - (k_{3i} + k_{4i}) \left[ \sqrt{k_{1i}^2 + (k_{3i} + k_{4i})^2} - \sqrt{(k_{1i} + k_{3i} + k_{4i})^2 + (k_{3i} + k_{4i})^2} \right] \right\}
\]

\[
e_{zi} = \frac{1}{3} \left\{ k_{3i} \left[ \sqrt{k_{1i}^2 + k_{2i}^2} - \sqrt{(k_{1i} + k_{3i} + k_{4i})^2 + k_{3i}^2} \right] - \left( k_{1i}^2 + k_{2i}^2 + k_{3i}^2 + 1 \right)^{\frac{3}{2}} \right\}
\]
\[ e_{zi} = \frac{1}{2} \left(T_{3z} - T_{6z} + T_{7z} + T_{8z} - k_{1z}^2(T_{3z} - T_{6z}) + (k_{1z} + k_{2z})^2(T_{3z} - T_{4z}) - k_{1z}^2(T_{1z} - T_{16z}) + (k_{3z} + k_{4z})^2(T_{15z} - T_{16z}) + 2k_{3z}[k_{1z}L_{15z} - (k_{1z} + k_{2z})L_{16z}]ight) \]

\[ e_{zi} = \frac{1}{2} \left(-T_{3z} + T_{2z} + T_{4z} + T_{9z} - T_{10z} - T_{11z} + T_{12z} + k_{2z}^2(T_{3z} - T_{4z}) + k_{3z}L_{15z} - (k_{1z} + k_{2z})L_{16z} - (k_{3z} + k_{4z})[k_{1z}L_{17z} - (k_{1z} + k_{2z})L_{18z}]ight) \]

where \( T_{13i} \), \( T_{16i} \), and \( L_{15i} - L_{18i} \) \((i = 1, 2, 3)\) are expressed as

\[ T_{13i} = \tan^{-1} \frac{k_{1i}}{k_{3i}\sqrt{k_{1i}^2 + k_{2i}^2 + 1}} \]

\[ T_{14i} = \tan^{-1} \frac{k_{1i} + k_{2i}}{k_{3i}(k_{1i} + k_{2i})^2 + k_{3i}^2 + 1} \]

\[ T_{15i} = \tan^{-1} \frac{k_{1i}}{(k_{3i} + k_{4i})\sqrt{(k_{1i} + k_{2i})^2 + (k_{3i} + k_{4i})^2 + 1}} \]

\[ T_{16i} = \tan^{-1} \frac{k_{1i} + k_{2i}}{(k_{3i} + k_{4i})\sqrt{(k_{1i} + k_{2i})^2 + (k_{3i} + k_{4i})^2 + 1}} \]

\[ L_{15i} = \ln \left| \frac{\sqrt{k_{1i}^2 + k_{3i}^2 + 1 + 1}}{k_{1i} + k_{3i}} \right| \]

\[ L_{16i} = \ln \left| \frac{\sqrt{(k_{1i} + k_{2i})^2 + k_{3i}^2 + 1 + 1}}{(k_{1i} + k_{2i})^2 + k_{3i}^2 + 1} \right| \]

\[ L_{17i} = \ln \left| \frac{\sqrt{k_{1i}^2 + (k_{3i} + k_{4i})^2 + 1 + 1}}{\sqrt{k_{1i}^2 + (k_{3i} + k_{4i})^2}} \right| \]

\[ L_{18i} = \ln \left| \frac{\sqrt{(k_{1i} + k_{2i})^2 + (k_{3i} + k_{4i})^2 + 1 + 1}}{(k_{1i} + k_{2i})^2 + (k_{3i} + k_{4i})^2} \right| \]

**Notation**

**The following symbols are used in this paper:**

- \( A_{ij} \) \((i, j = 1 - 6)\) = elastic moduli or elasticity constants;
- \( a \) = loads applied at a horizontal distance in the \( x \)-axis from the wall;
- \( c \) = loads applied at a horizontal distance in the \( y \)-axis from the wall;
- \( d_{11i} - d_{34i} \) = stress elementary functions for loading Case B (Appendix I);
- \( d_{11i'} - d_{34i'} \) = force integral functions for loading Case B (Appendix II);
- \( d_{11i} - d_{34i} \) = centroid integral functions for loading Case B (Appendix III);
- \( E, E', v, v', G' \) = elastic engineering constants of a cross-anisotropic backfill;
- \( e_{s1i} - e_{s4i} \) = stress elementary functions for loading Case C (Appendix I);
- \( e_{s1i} - e_{s4i} \) = force integral functions for loading Case C (Appendix II);
- \( e_{s1i} - e_{s4i} \) = centroid integral functions for loading Case C (Appendix III);
- \( H \) = height of the retaining wall;
- \( i \) = complex number \((= \sqrt{-1})\);
- \( k \) = coefficients;
- \( k_{1i} - k_{4i} \) = functions of \( a, l, c, w, H \), and \( u_i \) \((i = 1, 2, \) and \(3)\);
- \( l \) = length of the rectangular area load;
- \( m_1, m_2 \) = coefficients;
- \( n_{1i} - n_{4i} \) = functions of \( a, l, c, w, z \), and \( u_i \) \((i = 1, 2, \) and \(3)\);
- \( P \) = a horizontal point load (force);
- \( P_i \) = a horizontal finite line load (force per unit length);
- \( P_s \) = a horizontal uniform rectangular area load (force per unit area);
- \( P_{h} \) = lateral force due to loading Case B (Appendix II);
- \( P_{h} \) = lateral force due to loading Case A (Appendix II);
- \( P_{h} \) = lateral force due to loading Case C (Appendix II);
- \( P_{s} \) = stress elementary functions for loading Case A (Appendix I);
- \( P_{s} \) = force integral functions for loading Case A (Appendix II);
- \( P_{s} \) = centroid integral functions for loading Case A (Appendix III);
\[ Q = \text{a vertical point load (force);} \]
\[ Q^f = \text{a vertical finite line load (force per unit length);} \]
\[ Q^a = \text{a vertical uniform rectangular area load (force per unit area);} \]
\[ s, t = \text{coefficients;} \]
\[ u_1, u_2, u_3 = \text{roots of the characteristic equation;} \]
\[ w = \text{width of the rectangular area load;} \]
\[ \bar{z}_h = \text{centroid location due to loading Case B when } Q^h = 0 \text{ (Appendix III);} \]
\[ \bar{z}_v = \text{centroid location due to loading Case B when } P^v = 0 \text{ (Appendix III);} \]
\[ \bar{z}_w = \text{centroid location due to loading Case A when } Q^w = 0 \text{ (Appendix III);} \]
\[ \bar{z}^v = \text{centroid location due to loading Case B when } P^v = 0 \text{ (Appendix III);} \]
\[ \bar{z}^h = \text{centroid location due to loading Case C when } Q^h = 0 \text{ (Appendix III);} \]
\[ \bar{z}^w = \text{centroid location due to loading Case C when } P^w = 0 \text{ (Appendix III);} \]
\[ \sigma^p = \text{lateral stress due to loading Case B (Appendix I);} \]
\[ \sigma^v = \text{lateral stress due to loading Case A (Appendix I);} \]
\[ \sigma^w = \text{lateral stress due to loading Case C (Appendix I);} \]

References


